MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

PES 0024 – ESSENTIAL STATISTICS

(All sections / Groups)

3 MARCH 2020
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of NINE **printed** pages excluding cover page.
2. Attempt **ALL** FOUR questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided. All necessary working steps **MUST** be shown.
4. **Formula** is provided at the back of the question paper.
5. **Statistical table** is provided at the back of the question paper.

Question 1 (25 marks)

(a) A question in a survey asked students of MEA College about how often they visited the college's tutoring center in the last month. The participating students includes both full time and part time students. The results are shown below.

	Number of visits			
	One or fewer times (O)	Two to three times (T)	Four or more times (F)	Total
Full time student (FT)	23	130	97	250
Part time student (PT)	5	25	20	50
Total	28	155	117	300

Suppose that a surveyed student is a randomly selected. Determine:

(i) the probability that student is a part time student and visited the tutoring centre two to three times? **(3 marks)**

(ii) the probability that student visited the tutoring centre four or more times? **(3 marks)**

(iii) the probability that student visited the tutoring centre four or more times, given that the student is full time? **(4 marks)**

(iv) whether 'full time student' and students that 'visited the tutoring centre for four or more times' are statistically independent. Use appropriate answers from the previous parts in your justification. **(3 marks)**

(v) whether 'full time student' and 'part time student' are mutually exclusive events. State your justification. **(3 marks)**

Continued...

(b) The following shows the results of a survey on the types of exercise taken by a group of 100 students in MMU Cyberjaya.

65 run
48 swim
60 cycle
40 run and swim
30 swim and cycle
35 run and cycle
25 do all three

(i) Draw a Ven Diagram to represent these data.

(3 marks)

(ii) Find the probability that a randomly selected student from the survey swim but does not run.

(3 marks)

(iii) Find the probability that a randomly selected student from the survey takes at least two of these types of exercise.

(3 marks)

Question 2 (25 marks)

(a) The probability distribution table below describes results of the roadworthiness tests of Ford Focus cars that are 3 years old (based on the data from the Department of Transportation). The random variable x represents the number of cars that failed among six that were tested for roadworthiness.

x	P(x)
0	0.357
1	0.339
2	0.156
3	a
4	0.005
5	0.003
6	0.001

(i) Find the value of a. **(2 marks)**

(ii) Compute the mean for the numbers of cars that failed among the six cars that are tested. **(4 marks)**

(iii) Compute the standard deviation. **(4 marks)**

Continued...

(b) The time taken (in minutes) for a coffee machine to make a cup of coffee is represented by a continuous random variable X with the following density function below:

$$f(x) = \begin{cases} \frac{3x^2}{2}, & -1 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Show that the function *f(x)* is a valid probability function. **(4 marks)**

(ii) Find the probability that it takes more than $\frac{1}{2}$ but fewer than $\frac{4}{5}$ minutes for the coffee to be ready. **(4 marks)**

(iii) Calculate the mean and standard deviation of *f(x)*. **(7 marks)**

Question 3 (25 marks)

(a) Assume that Richter scale magnitudes of earthquakes are normally distributed with a mean of 1.184 and a standard deviation of 0.587.

(i) Earthquakes with magnitudes less than 2.000 are considered "micro earthquakes". What is the percentage of earthquakes fall into this category? **(4 marks)**

(ii) Earthquakes with magnitudes above 4.0 will cause indoor items to shake. What percentage of earthquakes fall into this category?

(4 marks)

(b) In "10 Grams Café", an average of 3 out of every 5 customers ask for plain water with their meal. If a random sample of 10 customers is selected, find the probability that:

(i) exactly 6 ask for water with their meal, **(4 marks)**
(ii) less than 9 ask for water with their meal, **(4 marks)**

Continued...

(c) Patients arrive at a hospital's accident and emergency department at random at a rate of 6 per hour.

(i) Find the probability that during any 90 minutes' period, the number of patients arriving at the hospital accident and emergency department is exactly 7? **(3 marks)**

(ii) Find the mean and standard deviation.

(3 marks)

(d) Continuous from part (c), a patient arrives at 11.30 a.m. Find the probability that the next patient arrives before 11.45 a.m.

(3 marks)

Question 4 (25 marks)

(a) A Boeing 757-200 ER airliner can carry 200 passengers and has doors with height of 72 in. Heights of men are normally distributed with a mean of 69.5 in. and a standard deviation of 2.4 in.

(i) Construct a 95% confidence interval estimate for the mean height of the passenger. Interpret the answer

(4 marks)

(ii) If a male passenger is randomly selected, find the probability that he can fit through the doorways without bending.

(4 marks)

(iii) If half of the 200 passengers are men, find the probability that the mean height of the 100 men is less than 72 in.

(4 marks)

Continued...

(b) Suppose that the population of interest consists of the five passengers having to answer a pop quiz to win a free ticket. The table below lists the passenger and their scores.

Player	Scores
Anne	3
Belle	2
Cathy	1
Donna	4
Enny	1

List all the possible samples of three selected passengers (without replacement) from this population and construct the sampling distribution of the sample mean.

(13 marks)

End of Page

Formula Sheet

Descriptive Statistics

		Population	Sample
Ungrouped Data			
Mean		$\mu = \frac{\sum x}{N}$	$\bar{x} = \frac{\sum x}{n}$
Variance		$\sigma^2 = \frac{\sum (x-\mu)^2}{N}$	$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$
	OR	$\sigma^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{N}}{N}$	$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$
Standard Deviation		$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$
Grouped data			
Mean		$\mu = \frac{\sum fm}{N}$	$\bar{x} = \frac{\sum fm}{n}$
Variance		$\sigma^2 = \frac{\sum f(m-\mu)^2}{N}$	$s^2 = \frac{\sum f(m-\bar{x})^2}{n-1}$
	OR	$\sigma^2 = \frac{\sum fm^2 - \frac{(\sum fm)^2}{N}}{N}$	
		$s^2 = \frac{\sum fm^2 - \frac{(\sum fm)^2}{n}}{n-1}$	
Standard Deviation		$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$
Median		$\frac{(n+1)^{th}}{2}$ observation of data	

Probability Distributions

Discrete

Binomial Distribution $\mu = np$ $\sigma = \sqrt{npq}$

Continuous

Normal Distribution $z = \frac{x - \mu}{\sigma}$

Sampling Distributions

Mean $\mu_{\bar{x}} = \mu$

Standard Deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

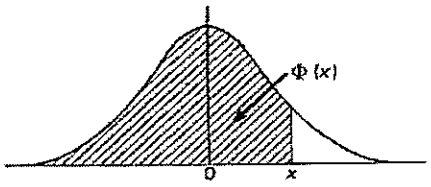
Estimation

Confidence Interval $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Random Variables	**Mean**	**Variance**
Discrete	$\sum x \cdot p(x)$	$\sum x^2 \cdot p(x) - \mu^2$
Continuous	$\int_{\infty}^{-\infty} x \cdot f(x)dx$	$\int_{\infty}^{-\infty} x^2 \cdot f(x)dx - \mu^2$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

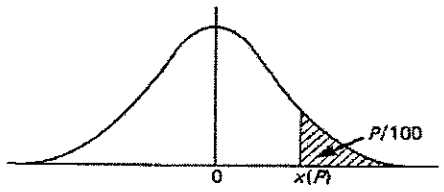
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172